THERMOELASTICITY OF FLEXIBLE MULTILAYER ANISOTROPIC SHELLS

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The three-dimensional problem of thermoelasticity for multilayer anisotropic shells of very simple geometric forms and with particular forms of the boundary conditions is solved in [1, 2]. Analysis of thermoelastic multilayer anisotropic shells of the Timoshenko type in the geometrically nonlinear formulations is discussed in [3]. We can evaluate the present status of the problem on the basis of the survey articles [4-7], from which we see that the problem of the analysis of geometrically nonlinear multilayer anisotropic shells that are subjected to thermomechanical loading has been very little studied. We shall examine in the geometrically nonlinear formulation thermoelastic multilayer anisotropic shells with account for the local effects on the basis of the Grigolyuk kinematic hypothesis [8]. The layers are oriented in the packet so that their axes of orthotropy do not coincide with the directions of the coordinate lines. We note that the general theory of multilayer anisotropic shells based on the Grigolyuk hypothesis has been constructed earlier without account for the temperature effects in [9, 10].

1. The mixed variational equation of thermoelasticity for a nonlinear anisotropic shell. We shall examine a thin shell of constant thickness h, consisting of N anisotropic layers. We take as the reference surface Π the inner surface of the shell, which we refer to the α_1, α_2 curvilinear orthogonal coordinates, measured along the principal lines of curvature. We measure the transverse coordinate z in the direction of increase of the outward normal to the reference surface. We introduce the following notations: h_k is the thickness of the k-th layer; δ_k is the distance from the reference surface to the "upper" bounding surface of the k-th layer ($\delta_0 = 0$); A_i are the Lamé parameters; k_i are the curvatures of the coordinate lines; $u_i^{(k)}(\alpha_1, \alpha_2, z)$, $u_3^{(k)}(\alpha_1, \alpha_2, z)$ are the tangential and normal displacements of the points of the k-th layer; $u_i(\alpha_1, \alpha_2)$, $w(\alpha_1, \alpha_2)$ are the tangential and normal displacements of the reference surface; $\beta_i^{(k)}(\alpha_1, \alpha_2)$ are the angles of rotation of the normal in the limits of the k-th layer; $\mu_i^{(k)}(\alpha_1, \alpha_2)$ are functions that characterize the distribution of the transverse tangential stresses through the thickness of the k-th layer; $\tau_i^{(k)}(\alpha_1, \alpha_2)$ are the transverse tangential stresses of the upper bounding surface of the k-th layer ($\tau_i^{(0)} = p_i^-$, $\tau_i^{(N)} = p_i^+$); p_i^-, q^- and p_i^+, q^+ are the components of the external surface loads, acting on the inner and outer surfaces of the shell. Here and in the following $i, j = 1, 2$; $k = 1, 2, ..., N$.

We shall introduce the deformation characteristics of the shell. We determine the tangential and transverse components of the strain tensor in the case of the very simple nonlinear variant of shell theory in the quadratic approximation [11] from the formulas

$$\varepsilon_{11}^{(k)} = \frac{1}{1 + k_1 z}\left(\frac{1}{A_1}\frac{\partial u_1^{(k)}}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} u_2^{(k)} + k_1 u_3^{(k)}\right) + \frac{1}{2}(\theta_1^{(k)})^2$$

$$\varepsilon_{12}^{(k)} = \frac{1}{1 + k_1 z}\left(\frac{1}{A_1}\frac{\partial u_2^{(k)}}{\partial \alpha_1} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} u_1^{(k)}\right) + \tag{1.1}$$

$$+ \frac{1}{1 + k_2 z}\left(\frac{1}{A_2}\frac{\partial u_1^{(k)}}{\partial \alpha_2} - \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} u_2^{(k)}\right) + \theta_1^{(k)}\theta_2^{(k)}$$

$$\varepsilon_{13}^{(k)} = \partial u_1^{(k)}/\partial z + \theta_1^{(k)}, \quad \varepsilon_{33}^{(k)} = \partial u_3^{(k)}/\partial z$$

$$\theta_1^{(k)} = \frac{1}{1 + k_1 z}\left(\frac{1}{A_1}\frac{\partial u_3^{(k)}}{\partial \alpha_1} - k_1 u_1^{(k)}\right) \quad (1 \rightleftarrows 2) \tag{1.1}$$

We obtain the equations of equilibrium of the multilayer shell from the mixed Reissner variational principle [12]:

$$\delta\,(J - A) = 0 \tag{1.2}$$

which discloses a natural way to reduce the three-dimensional problems of elasticity theory to the two-dimensional problems of shell theory. Here A is the work of the external loads, and we represent the functional J in the form

$$J = \iint_\Pi \sum_{k=1}^{N} \int_{\delta_{k-1}}^{\delta_k} [\sigma_{11}^{(k)}\varepsilon_{11}^{(k)} + \sigma_{22}^{(k)}\varepsilon_{22}^{(k)} + \sigma_{12}^{(k)}\varepsilon_{12}^{(k)} + \sigma_{33}^{(k)}\varepsilon_{33}^{(k)} + \sigma_{13}^{(k)}\varepsilon_{13}^{(k)} + \tag{1.3}$$

$$+ \sigma_{23}^{(k)}\varepsilon_{23}^{(k)} + G_k]\, A_1 A_2\, (1 + k_1 z)\,(1 + k_2 z)\, dz\, d\alpha_1\, d\alpha_2$$

where G_k is the Gibbs thermodynamic potential of the k-th layer [13]:

$$-G_k = \tfrac{1}{2} a_{11}^{(k)} (\sigma_{11}^{(k)})^2 + a_{12}^{(k)}\sigma_{11}^{(k)}\sigma_{22}^{(k)} + \underline{a_{13}^{(k)}\sigma_{11}^{(k)}\sigma_{33}^{(k)}} + a_{16}^{(k)}\sigma_{11}^{(k)}\sigma_{12}^{(k)} +$$

$$+ \tfrac{1}{2} a_{22}^{(k)} (\sigma_{22}^{(k)})^2 + \underline{a_{23}^{(k)}\sigma_{22}^{(k)}\sigma_{33}^{(k)}} + a_{26}^{(k)}\sigma_{22}^{(k)}\sigma_{12}^{(k)} + \tfrac{1}{2}\underline{a_{33}^{(k)} (\sigma_{33}^{(k)})^2} +$$

$$+ a_{36}^{(k)}\sigma_{33}^{(k)}\sigma_{12}^{(k)} + \tfrac{1}{2} a_{66}^{(k)} (\sigma_{12}^{(k)})^2 + \tfrac{1}{2} a_{44}^{(k)} (\sigma_{23}^{(k)})^2 + a_{45}^{(k)}\sigma_{23}^{(k)}\sigma_{13}^{(k)} + \tag{1.4}$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$+ \tfrac{1}{2} a_{55}^{(k)} (\sigma_{13}^{(k)})^2 + (\alpha_1^{(k)}\sigma_{11}^{(k)} + \alpha_2^{(k)}\sigma_{22}^{(k)} + \alpha_3^{(k)}\sigma_{33}^{(k)} + \alpha_6^{(k)}\sigma_{12}^{(k)} +$$

$$+ \alpha_4^{(k)}\sigma_{23}^{(k)} + \alpha_5^{(k)}\sigma_{13}^{(k)})\,\theta + \tfrac{1}{2} c_\sigma^{(k)}\, \theta^2/T_0$$

Here $a_{11}^{(k)}, \ldots, a_{66}^{(k)}$ are the compliances of the k-th layer; $\alpha_1^{(k)}, \alpha_2^{(k)}, \alpha_3^{(k)}$ are the coefficients of linear thermal expansion of the k-th layer; $\alpha_4^{(k)}, \alpha_5^{(k)}, \alpha_6^{(k)}$ are the coefficients of thermal displacement of the k-th layer; $c_\sigma^{(k)}$ is the heat capacity at constant stress of the k-th layer; $\theta = T - T_0$ is the temperature increase relative to the initial state T_0.

We shall assume that the normal stresses $\sigma_{33}^{(k)}$, acting on the areas that are parallel to the areas of the reference surface, are negligibly small in comparison with the other components of the stress tensor; therefore the underlined terms in the expression for the Gibbs thermodynamic potential (1.4) can be dropped. However the term $\alpha_3^{(k)}\sigma_{33}^{(k)}$ must be retained, since the coefficient of linear thermal expansion in the direction normal to the reference surface for the composite materials may exceed the values of the coefficients of linear thermal expansion in the tangential directions.

Introducing the potential G_k from (1.4) and the deformations (1.1) into the mixed Reissner variational principle (1.2), (1.3) and varying independently the displacements and the stresses, we obtain with account for the generalized formulas of integration by parts the variational equation

$$\iint_\Pi \sum_{k=1}^{N} \int_{\delta_{k-1}}^{\delta_k} \{-L_1^{(k)}\delta u_1^{(k)} - L_2^{(k)}\delta u_2^{(k)} - L_3^{(k)}\delta u_3^{(k)} + [(\varepsilon_{11}^{(k)} - a_{11}^{(k)}\sigma_{11}^{(k)} -$$

$$- a_{12}^{(k)}\sigma_{22}^{(k)} - a_{16}^{(k)}\sigma_{12}^{(k)} - \alpha_1^{(k)}\theta)\,\delta\sigma_{11}^{(k)} + (\varepsilon_{22}^{(k)} - a_{12}^{(k)}\sigma_{11}^{(k)} - a_{22}^{(k)}\sigma_{22}^{(k)} - \tag{1.5}$$

$$- a_{26}^{(k)}\sigma_{12}^{(k)} - \alpha_2^{(k)}\theta)\,\delta\sigma_{22}^{(k)} + (\varepsilon_{33}^{(k)} - \alpha_3^{(k)}\theta)\,\delta\sigma_{33}^{(k)} + (\varepsilon_{12}^{(k)} - a_{16}^{(k)}\sigma_{11}^{(k)} -$$

$$- a_{26}^{(k)}\sigma_{22}^{(k)} - a_{66}^{(k)}\sigma_{12}^{(k)} - \alpha_6^{(k)}\theta)\,\delta\sigma_{12}^{(k)} + (\varepsilon_{23}^{(k)} - a_{44}^{(k)}\sigma_{23}^{(k)} - a_{45}^{(k)}\sigma_{13}^{(k)} -$$

$$- \alpha_4^{(k)}\theta)\,\delta\sigma_{23}^{(k)} + (\varepsilon_{13}^{(k)} - a_{45}^{(k)}\sigma_{23}^{(k)} - a_{55}^{(k)}\sigma_{13}^{(k)} - \alpha_5^{(k)}\theta)\,\delta\sigma_{13}^{(k)}]\, H_1 H_2\}\, dz\, d\alpha_1\, d\alpha_2 +$$

$$
\begin{aligned}
&+\iint_{\Pi} [(\sigma_{13}^{(N)} - p_1^+)\, \delta u_1^{(N)} + (\sigma_{23}^{(N)} - p_2^+)\, \delta u_2^{(N)} + (\sigma_{33}^{(N)} - q^+)\, \delta u_3^{(N)}]\, H_1 H_2 \Big|_{z=\delta_N} d\alpha_1 d\alpha_2 - \\
&- \iint_{\Pi} [(\sigma_{13}^{(1)} - p_1^-)\, \delta u_1^{(1)} + (\sigma_{23}^{(1)} - p_2^-)\, \delta u_2^{(1)} + (\sigma_{33}^{(1)} - q^-)\, \delta u_3^{(1)}]\, H_1 H_2 \Big|_{z=\delta_0} d\alpha_1 d\alpha_2 + \\
&+ \iint_{\Pi} \sum_{n=1}^{N-1} [(\sigma_{13}^{(n)}\delta u_1^{(n)} - \sigma_{13}^{(n+1)}\delta u_1^{(n+1)}) + (\sigma_{23}^{(n)}\delta u_2^{(n)} - \sigma_{23}^{(n+1)}\delta u_2^{(n+1)}) + (\sigma_{33}^{(n)}\delta u_3^{(n)} - \\
&- \sigma_{33}^{(n+1)}\delta u_3^{(n+1)})\,]\, H_1 H_2 \Big|_{z=\delta_n} d\alpha_1 d\alpha_2 + \int_{\Gamma_2^+} \sum_{k=1}^{N} \int_{\delta_{k-1}}^{\delta_k} [(\sigma_{11}^{(k)} - X_1^+)\, \delta u_1^{(k)} + \\
&+ (\sigma_{12}^{(k)} - X_2^+)\, \delta u_2^{(k)} + (S_{13}^{(k)} - X_3^+)\, \delta u_3^{(k)}]\, H_2 \Big|_{\alpha_1=\alpha_1^+} dz d\alpha_2 - \int_{\Gamma_2^-} \sum_{k=1}^{N} \int_{\delta_{k-1}}^{\delta_k} [(\sigma_{11}^{(k)} - X_1^-)\, \delta u_1^{(k)} + \qquad (1.5) \\
&+ (\sigma_{12}^{(k)} - X_2^-)\, \delta u_2^{(k)} + (S_{13}^{(k)} - X_3^-)\, \delta u_3^{(k)}]\, H_2 \Big|_{\alpha_1=\alpha_1^-} dz d\alpha_2 + \\
&+ \int_{\Gamma_1^+} \sum_{k=1}^{N} \int_{\delta_{k-1}}^{\delta_k} [(\sigma_{12}^{(k)} - Y_1^+)\, \delta u_1^{(k)} + (\sigma_{22}^{(k)} - Y_2^+)\, \delta u_2^{(k)} + (S_{23}^{(k)} - Y_3^+)\, \delta u_3^{(k)}]\, H_1 \Big|_{\alpha_2=\alpha_2^+} dz d\alpha_1 - \\
&- \int_{\Gamma_1^-} \sum_{k=1}^{N} \int_{\delta_{k-1}}^{\delta_k} [(\sigma_{12}^{(k)} - Y_1^-)\, \delta u_1^{(k)} + (\sigma_{22}^{(k)} - Y_2^-)\, \delta u_2^{(k)} + \\
&+ (S_{23}^{(k)} - Y_3^-)\, \delta u_3^{(k)}]\, H_1 \Big|_{\alpha_2=\alpha_2^-} dz d\alpha_1 = 0
\end{aligned}
$$

where $X_i^{\pm}, X_3^{\pm}$ and $Y_i^{\pm}, Y_3^{\pm}$ are the external surface loads, acting on the side surfaces of the shell $\alpha_1 = \alpha_1^{\pm}$ and $\alpha_2 = \alpha_2^{\pm}$; $L_i^{(k)} L_3^{(k)}$ are the nonlinear differential operators of the three-dimensional theory of elasticity [11], corresponding to the introduced deformation field (1.1):

$$
\begin{aligned}
L_1^{(k)} &= \frac{\partial (H_2 \sigma_{11}^{(k)})}{\partial \alpha_1} - \frac{H_1}{A_1} \frac{\partial A_2}{\partial \alpha_1} \sigma_{22}^{(k)} + \frac{\partial (H_1 \sigma_{12}^{(k)})}{\partial \alpha_2} + \frac{H_2}{A_2} \frac{\partial A_1}{\partial \alpha_2} \sigma_{12}^{(k)} + \frac{\partial (H_1 H_2 \sigma_{13}^{(k)})}{\partial z} + k_1 A_1 H_2 S_{13}^{(k)} \\
L_3^{(k)} &= \frac{\partial (H_2 S_{13}^{(k)})}{\partial \alpha_1} + \frac{\partial (H_1 S_{23}^{(k)})}{\partial \alpha_2} + \frac{\partial (H_1 H_2 \sigma_{33}^{(k)})}{\partial z} - k_1 A_1 H_2 \sigma_{11}^{(k)} - k_2 A_2 H_1 \sigma_{22}^{(k)} \qquad (1.6) \\
S_{13}^{(k)} &= \sigma_{13}^{(k)} + \theta_1^{(k)} \sigma_{11}^{(k)} + \theta_2^{(k)} \sigma_{12}^{(k)}, \quad H_1 = A_1 (1 + k_1 z) \ \ (1 \rightleftarrows 2)
\end{aligned}
$$

2. Resolving equations of the multilayer anisotropic shell of moderate thickness. In formulating the theory of multilayer anisotropic shells with account for the local effects, we shall use the Grigolyuk kinematic hypothesis on the piecewise-linear law of variation of the tangential displacements through the thickness of the shell [12]:

$$
u_i^{(k)} = u_i + \sum_{n=1}^{N} \pi_{kn} \beta_i^{(n)} + (z - \delta_{k-1})\, \beta_i^{(k)}
\qquad (2.1)
$$

$$
\pi_{kn} = \begin{cases} h_n & \text{for} \quad k > n \\ 0 & \text{for} \quad k \le n \end{cases}
$$

and the kinematic hypothesis [14] on the nonlinear dependence of the normal displacements on the transverse coordinate in the limits of the k-th layer

$$
u_3^{(k)} = w + \gamma_0^{(k)}, \quad \gamma_0^{(k)} = \sum_{n=1}^{k-1} \alpha_3^{(n)} \int_{\delta_{n-1}}^{\delta_n} \theta dz + \alpha_3^{(k)} \int_{\delta_{k-1}}^{z} \theta dz \qquad (2.2)
$$

We take for the transverse tangential stresses the independent approximation [12]

$$\sigma_{i3}^{(k)} = \overline{M}_k(z)\,\tau_i^{(k-1)} + M_k(z)\,\tau_i^{(k)} + f_k(z)\,\mu_i^{(k)}$$

$$M_k(z) = \frac{1}{h_k}(z - \delta_{k-1}), \quad \overline{M}_k(z) = 1 - M_k(z), \quad f_k(z) = \frac{6}{h_k}\overline{M}_k(z)\,M_k(z) \tag{2.3}$$

We introduce the displacements and the transverse tangential stresses in accordance with formulas (2.1), (2.2), (2.3) into the variational equation (1.5) and, equating to zero the expressions preceding the variations of the independent variables u_i, $\beta_i^{(k)}$, w, $\sigma_{ij}^{(k)}$, $\sigma_{33}^{(k)}$, $\mu_i^{(k)}$, $\tau_i^{(n)}(n = 1, 2, ..., N - 1)$, obtain:

the elasticity relations for the tangential stresses and strains

$$\begin{aligned}
\varepsilon_{11}^{(k)} &= a_{11}^{(k)}\sigma_{11}^{(k)} + a_{12}^{(k)}\sigma_{22}^{(k)} + a_{16}^{(k)}\sigma_{12}^{(k)} + \alpha_1^{(k)}\theta \\
\varepsilon_{22}^{(k)} &= a_{12}^{(k)}\sigma_{11}^{(k)} + a_{22}^{(k)}\sigma_{22}^{(k)} + a_{26}^{(k)}\sigma_{12}^{(k)} + \alpha_2^{(k)}\theta \\
\varepsilon_{12}^{(k)} &= a_{16}^{(k)}\sigma_{11}^{(k)} + a_{26}^{(k)}\sigma_{22}^{(k)} + a_{66}^{(k)}\sigma_{12}^{(k)} + \alpha_6^{(k)}\theta
\end{aligned} \tag{2.4}$$

the equations for determining the transverse normal strains

$$\varepsilon_{33}^{(k)} = \alpha_3^{(k)}\theta \tag{2.5}$$

which are identically satisfied in accordance with formulas (1.1), (2.2);

the integral elasticity relations for the transverse tangential stresses and strains

$$\begin{aligned}
&\int_{\delta_{k-1}}^{\delta_k} \Lambda_i^{(k)} f_k(z)\,dz = 0 \quad (k = 1, 2, \ldots, N) \\
&\int_{\delta_{n-1}}^{\delta_n} \Lambda_i^{(n)} M_n(z)\,dz + \int_{\delta_n}^{\delta_{n+1}} \Lambda_i^{(n+1)}\overline{M}_{n+1}(z)\,dz = 0 \quad (n = 1, 2, \ldots, N-1) \\
&\Lambda_1^{(k)} = \varepsilon_{13}^{(k)} - a_{45}^{(k)}\sigma_{23}^{(k)} - a_{55}^{(k)}\sigma_{13}^{(k)} - \alpha_5^{(k)}\theta \quad (1 \rightleftarrows 2,\ 4 \rightleftarrows 5)
\end{aligned} \tag{2.6}$$

the $2N + 2$ equations of equilibrium of the multilayer shell in the forces and moments

$$\begin{aligned}
&\sum_{k=1}^{N} \int_{\delta_{k-1}}^{\delta_k} L_i^{(k)}\,dz = 0 \\
&\int_{\delta_{k-1}}^{\delta_k} L_i^{(k)}(z - \delta_{k-1})\,dz + \sum_{n=1}^{N} \pi_{nk} \int_{\delta_{n-1}}^{\delta_n} L_i^{(n)} dz = 0
\end{aligned} \tag{2.7}$$

the N nonlinear equations of the three-dimensional theory of elasticity

$$L_3^{(k)} = 0 \tag{2.8}$$

the boundary conditions on the inner surface of the shell $z = \delta_0$:

$$\sigma_{i3}^{(1)} = p_i^-, \quad \sigma_{33}^{(1)} = q^- \tag{2.9}$$

the first two of which are satisfied in accordance with (2.3);

the boundary conditions on the outer surface of the shell $z = \delta_N$:

$$\sigma_{i3}^{(N)} = p_i^+, \quad \sigma_{33}^{(N)} = q^+ \tag{2.10}$$

the first two of which are satisfied in accordance with (2.3);

the continuity conditions on the interfaces of the layers $z = \delta_n$:

$$\sigma_{\alpha 3}^{(n)} = \sigma_{\alpha 3}^{(n+1)}, \quad \sigma_{33}^{(n)} = \sigma_{33}^{(n+1)} \quad (n = 1, 2, \ldots, N-1) \tag{2.11}$$

the first $2N - 2$ of which are satisfied in accordance with (2.3);

the boundary conditions on the side surfaces $\alpha_1 = \alpha_1^+$, $\alpha_1 = \alpha_1^-$:

$$\begin{gathered}(T_{11} - T_{11}^*)\,\delta u_1 = 0, \quad (T_{12} - T_{12}^*)\,\delta u_2 = 0, \quad (T_{13} - Q_1^*)\,\delta w = 0 \\ (\Phi_{11}^{(k)} - \Phi_{11}^{(k)*})\,\delta\beta_1^{(k)} = 0, \quad (\Phi_{12}^{(k)} - \Phi_{12}^{(k)*})\,\delta\beta_2^{(k)} = 0\end{gathered} \tag{2.12}$$

the boundary conditions on the side surfaces $\alpha_2 = \alpha_2^+$, $\alpha_2 = \alpha_2^-$, obtained from the boundary conditions (2.12) by permuting the indexes $(1 \rightleftarrows 2)$.

In the relations (2.12) we have introduced the classical forces and the generalized moments using the formulas

$$\begin{gathered}T_{rm} = \sum_{k=1}^{N} T_{rm}^{(k)}, \quad Q_1 = \sum_{k=1}^{N} Q_1^{(k)} \\ \Phi_{rm}^{(k)} = M_{rm}^{(k)} - \delta_{k-1} T_{rm}^{(k)} + \sum_{n=1}^{N} \pi_{nk} T_{rm}^{(n)} \quad (1 \rightleftarrows 2;\ r, m = 1, 2, 3) \\ T_{11}^{(k)} = \int_{\delta_{k-1}}^{\delta_k} \sigma_{11}^{(k)} (1 + k_2 z)\, dz, \quad T_{12}^{(k)} = \int_{\delta_{k-1}}^{\delta_k} \sigma_{12}^{(k)} (1 + k_2 z)\, dz \\ T_{13}^{(k)} = \int_{\delta_{k-1}}^{\delta_k} S_{13}^{(k)} (1 + k_2 z)\, dz, \quad Q_1^{(k)} = \int_{\delta_{k-1}}^{\delta_k} \sigma_{13}^{(k)} (1 + k_2 z)\, dz \\ M_{11}^{(k)} = \int_{\delta_{k-1}}^{\delta_k} \sigma_{11}^{(k)} z (1 + k_2 z)\, dz, \quad M_{12}^{(k)} = \int_{\delta_{k-1}}^{\delta_k} \sigma_{12}^{(k)} z (1 + k_2 z)\, dz \\ M_{13}^{(k)} = \int_{\delta_{k-1}}^{\delta_k} S_{13}^{(k)} z (1 + k_2 z)\, dz \quad (1 \rightleftarrows 2)\end{gathered} \tag{2.13}$$

With account for the notations (2.13), we can write the equations of equilibrium of the multilayer shell of moderate thickness (2.7) as:

$$\begin{gathered}\frac{\partial (A_2 T_{11})}{\partial \alpha_1} - \frac{\partial A_2}{\partial \alpha_1} T_{22} + \frac{\partial (A_1 T_{21})}{\partial \alpha_2} + \frac{\partial A_1}{\partial \alpha_2} T_{12} + A_1 A_2 k_1 T_{13} = \\ = A_1 A_2 (p_1^- - p_1^+) \quad (1 \rightleftarrows 2)\end{gathered} \tag{2.14}$$

$$\begin{gathered}\frac{\partial (A_2 \Phi_{11}^{(k)})}{\partial \alpha_1} - \frac{\partial A_2'}{\partial \alpha_1} \Phi_{22}^{(k)} + \frac{\partial (A_1 \Phi_{21}^{(k)})}{\partial \alpha_2} + \frac{\partial A_1}{\partial \alpha_2} \Phi_{12}^{(k)} - A_1 A_2 (Q_1^{(k)} - k_1 \Phi_{13}^{(k)}) = \\ = -A_1 A_2 h_k p_1^+ \quad (1 \rightleftarrows 2)\end{gathered} \tag{2.15}$$

We integrate the equation of three-dimensional elasticity theory (2.8) with respect to the z coordinate with account for the continuity conditions (2.11) and the boundary conditions (2.9). As a result we obtain

$$\begin{gathered}\sigma_{33}^{(k)} = q^- - \frac{1}{H_1 H_2} \Big(\frac{\partial (A_2 T_{13}^{(k)} [z])}{\partial \alpha_1} + \frac{\partial (A_1 T_{23}^{(k)} [z])}{\partial \alpha_2} - \\ - A_1 A_2 (k_1 T_{11}^{(k)} [z] + k_2 T_{22}^{(k)} [z]) \Big)\end{gathered} \tag{2.16}$$

$$T_{11}^{(k)}[z] = \sum_{n=1}^{k-1} \int_{\delta_{n-1}}^{\delta_n} \sigma_{11}^{(n)} (1 + k_2 z)\, dz + \int_{\delta_{k-1}}^{z} \sigma_{11}^{(k)} (1 + k_2 z)\, dz$$

$$T_{13}^{(k)}[z] = \sum_{n=1}^{k-1} \int_{\delta_{n-1}}^{\delta_n} S_{13}^{(n)} (1 + k_2 z)\, dz + \int_{\delta_{k-1}}^{z} S_{13}^{(k)} (1 + k_2 z)\, dz \quad (1 \rightleftarrows 2) \tag{2.17}$$

Satisfying the boundary conditions (2.10) and considering the equalities $T_{11}^{(N)}[\delta_N] = T_{11}$, $T_{13}^{(N)}[\sigma_N] = T_{13}(1 \rightleftarrows 2)$, we obtain the following equation of equilibrium of the multilayer shell

$$\partial (A_2 T_{13})/\partial \alpha_1 + \partial (A_1 T_{23})/\partial \alpha_2 - A_1 A_2 (k_1 T_{11} + K_2 T_{22}) = A_1 A_2 (q^- - q^+) \tag{2.18}$$

3. Resolving equations of the thin multilayer shell. Introducing the displacements (2.1), (2.2) into the deformation expressions (1.1), neglecting the terms $k_i z$ in comparison with unity, and assuming that the functions $\gamma_{i3}^{(k)}$ have a small index of variability with respect to the coordinate α_i, we obtain

$$\varepsilon_{ij}^{(k)} = E_{ij} + \sum_{n=1}^{N} \pi_{kn} K_{ij}^{(n)} + (z - \delta_{k-1}) K_{ij}^{(k)} + \delta_{ij} k_i \gamma_{i3}^{(k)}$$

$$\varepsilon_{i3}^{(k)} = \beta_i^{(k)} - \theta_i, \quad \varepsilon_{33}^{(k)} = \alpha_3^{(k)} \theta \tag{3.1}$$

where δ_{ij} is the Kronecker symbol, E_{ij} are the deformations of the reference surface:

$$E_{11} = \varepsilon_1 + \theta_1^2/2, \quad E_{12} = \omega_1 + \omega_2 + \theta_1 \theta_2$$

$$K_{11}^{(k)} = \varkappa_1^{(k)}, \quad K_{12}^{(k)} = \xi_1^{(k)} + \xi_2^{(k)}$$

$$\varepsilon_1 = \frac{1}{A_1}\frac{\partial u_1}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} u_2 + k_1 w, \quad \omega_1 = \frac{1}{A_1}\frac{\partial u_2}{\partial \alpha_1} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} u_1$$

$$\varkappa_1^{(k)} = \frac{1}{A_1}\frac{\partial \beta_1^{(k)}}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} \beta_2^{(k)}, \quad \xi_1^{(k)} = \frac{1}{A_1}\frac{\partial \beta_2^{(k)}}{\partial \alpha_1} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} \beta_1^{(k)}$$

$$\theta_1 = k_1 u_1 - \frac{1}{A_1}\frac{\partial w}{\partial \alpha_1} \quad (1 \rightleftarrows 2)$$

If we again use the assumption that the shell is thin-walled, taking the simplified formulas for the forces and moments

$$T_{ij}^{(k)} = \int_{\delta_{k-1}}^{\delta_k} \sigma_{ij}^{(k)} dz, \quad M_{ij}^{(k)} = \int_{\delta_{k-1}}^{\delta_k} \sigma_{ij}^{(k)} z\, dz, \quad Q_i^{(k)} = \int_{\delta_{k-1}}^{\delta_k} \sigma_{i3}^{(k)} dz$$

$$T_{13} = N_1, \quad N_1 = Q_1 - \theta_1 T_{11} - \theta_2 T_{12} \ (1 \rightleftarrows 2) \tag{3.2}$$

and in equations (2.15) we further neglect the generalized moments from the shearing stresses $\Phi_{13}^{(k)}$, then in this case the equations of equilibrium (2.14), (2.15), (2.18) coincide in their form with the corresponding equations of equilibrium of thin multilayer shell theory based on the Grigolyuk kinematic hypothesis [15], obtained without account for the thermal stresses. In their physical sense, however, these are different equations and they differ first of all in the presence in the formulas for the forces and moments of the terms corresponding to account for the thermal actions.

We rewrite the elasticity relations (2.4) in matrix form, after first solving them for the stresses

$$\left\| \begin{matrix} \sigma_{11}^{(k)} \\ \sigma_{22}^{(k)} \\ \sigma_{12}^{(k)} \end{matrix} \right\| = b^{(k)} \left\| \begin{matrix} \varepsilon_{11}^{(k)} \\ \varepsilon_{22}^{(k)} \\ \varepsilon_{12}^{(k)} \end{matrix} \right\| - \theta \left\| \begin{matrix} \eta_1^{(k)} \\ \eta_2^{(k)} \\ \eta_3^{(k)} \end{matrix} \right\|, \quad b^{(k)} = \left\| \begin{matrix} b_{11}^{(k)} & b_{12}^{(k)} & b_{16}^{(k)} \\ b_{12}^{(k)} & b_{22}^{(k)} & b_{26}^{(k)} \\ b_{16}^{(k)} & b_{26}^{(k)} & b_{66}^{(k)} \end{matrix} \right\| \tag{3.3}$$

The formulas for determining the elements of the matrix of the tangential stiffnesses of the k-th layer $\mathrm{b}^{(k)}$ are presented in [12]; $\eta_1^{(k)}$, $\eta_2^{(k)}$, $\eta_6^{(k)}$ are the tangential components of the thermal stress tensor with constrained deformations:

$$\eta_1^{(k)} = b_{11}^{(k)}\alpha_1^{(k)} + b_{12}^{(k)}\alpha_2^{(k)} + b_{16}^{(k)}\alpha_6^{(k)}$$
$$\eta_2^{(k)} = b_{12}^{(k)}\alpha_1^{(k)} + b_{22}^{(k)}\alpha_2^{(k)} + b_{26}^{(k)}\alpha_6^{(k)}$$
$$\eta_6^{(k)} = b_{16}^{(k)}\alpha_1^{(k)} + b_{26}^{(k)}\alpha_2^{(k)} + b_{66}^{(k)}\alpha_6^{(k)}$$

Substituting the expressions (3.3) into formulas (2.13) with account for the deformation relations (3.1) and the formulas (3.2), we obtain the expressions connecting the forces and moments with the deformation characteristics of the shell

$$\begin{Vmatrix} T_{11} \\ T_{22} \\ T_{12} \end{Vmatrix} = \mathrm{A} \begin{Vmatrix} E_{11} \\ E_{22} \\ E_{12} \end{Vmatrix} + \sum_{n=1}^{N} \mathrm{D}^{(n)} \begin{Vmatrix} K_{11}^{(n)} \\ K_{22}^{(n)} \\ K_{12}^{(n)} \end{Vmatrix} + \begin{Vmatrix} T_{\theta 1} \\ T_{\theta 2} \\ T_{\theta 6} \end{Vmatrix}$$

$$\begin{Vmatrix} \Phi_{11}^{(k)} \\ \Phi_{22}^{(k)} \\ \Phi_{12}^{(k)} \end{Vmatrix} = \mathrm{D}^{(k)} \begin{Vmatrix} E_{11} \\ E_{22} \\ E_{12} \end{Vmatrix} + \sum_{n=1}^{N} \mathrm{F}^{(kn)} \begin{Vmatrix} K_{11}^{(n)} \\ K_{22}^{(n)} \\ K_{12}^{(n)} \end{Vmatrix} + \begin{Vmatrix} \Phi_{\theta 1}^{(k)} \\ \Phi_{\theta 2}^{(k)} \\ \Phi_{\theta 6}^{(k)} \end{Vmatrix} \tag{3.4}$$

$$\mathrm{A} = \sum_{n=1}^{N} \mathrm{A}^{(n)}, \quad \mathrm{D}^{(k)} = \mathrm{B}^{(k)} - \delta_{k-1}\mathrm{A}^{(k)} + \sum_{n=1}^{N} \pi_{nk}\mathrm{A}^{(n)}$$

$$\mathrm{F}^{(kn)} = \delta_{kn}\,[\mathrm{C}^{(n)} - \delta_{n-1}\mathrm{B}^{(n)} - \delta_{n-1}\,(\mathrm{B}^{(n)} - \delta_{n-1}\mathrm{A}^{(n)})\,] + \pi_{kn}\,(\mathrm{B}^{(k)} - \delta_{k-1}\mathrm{A}^{(k)}) +$$

$$+ \pi_{nk}\,(\mathrm{B}^{(n)} - \delta_{n-1}\mathrm{A}^{(n)}) + \sum_{m=1}^{N} \pi_{mk}\pi_{mn}\mathrm{A}^{(m)}$$

$$\mathrm{A}^{(k)} = (\delta_k - \delta_{k-1})\,\mathrm{b}^{(k)}, \quad \mathrm{B}^{(k)} = 1/2\,(\delta_k^2 - \delta_{k-1}^2)\,\mathrm{b}^{(k)} \quad \mathrm{C}^{(k)} = 1/3\,(\delta_k^3 - \delta_{k-1}^3)\,\mathrm{b}^{(k)}$$

Here A, $\mathrm{D}^{(k)}$, $\mathrm{F}^{(kn)}$ are the stiffness matrices, and the integral characteristics of the temperature field have the form

$$T_{\theta m}^{(k)} = \int_{\delta_{k-1}}^{\delta_k} t_{\theta m}^{(k)} dz, \quad M_{\theta m}^{(k)} = \int_{\delta_{k-1}}^{\delta_k} t_{\theta m}^{(k)}\, z dz$$

$$\Phi_{\theta m}^{(k)} = M_{\theta m}^{(k)} - \delta_{k-1}T_{\theta m}^{(k)} + \sum_{n=1}^{N} \pi_{nk}T_{\theta m}^{(n)}, \quad T_{\theta m} = \sum_{k=1}^{N} T_{\theta m}^{(k)}$$

$$t_{\theta m}^{(k)} = d_m^{(k)}\gamma_0^{(k)} - \eta_m^{(k)}\theta \quad (m = 1, 2, 6), \quad d_1^{(k)} = k_1 b_{11}^{(k)} + k_2 b_{12}^{(k)}$$

$$d_2^{(k)} = k_1 b_{12}^{(k)} + k_2 b_{22}^{(k)}, \quad d_6^{(k)} = k_1 b_{16}^{(k)} + k_2 b_{26}^{(k)}$$

Introducing the transverse tangential stresses (2.3) into the integral elasticity relations (2.6) and excluding the functions $\mu_i^{(k)}$, we obtain the system of linear algebraic equations relative to the interlayer tangential stresses

$$\mathrm{G} \cdot \mathrm{U} = \mathrm{V}$$

$$\mathrm{U} = \|\tau_1^{(1)}, \tau_1^{(2)}, \ldots, \tau_1^{(N-1)}, \tau_2^{(1)}, \tau_2^{(2)}, \ldots, \tau_2^{(N-1)}\|^T \tag{3.5}$$

$$g_{mm} = -3\,(h_m a_{55}^{(m)} + h_{m+1} a_{55}^{(m+1)}), \quad g_{N-1+m,\,N-1+m} = -3\,(h_m a_{44}^{(m)} + h_{m+1} a_{44}^{(m+1)})$$

$$g_{m,\,N-1+m} = g_{N-1+m,\,m} = -3\,(h_m a_{45}^{(m)} + h_{m+1} a_{45}^{(m+1)})$$

$$g_{n,n+1} = g_{n+1,n} = h_{n+1}a_{55}^{(n+1)}, \quad g_{N-1+n,N+n} = g_{N+n,N-1+n} = h_{n+1}a_{44}^{(n+1)}$$

$$g_{n,N+n} = g_{N+n,n} = g_{n+1,N-1+n} = g_{N-1+n,n+1} = h_{n+1}a_{45}^{(n+1)}$$

$$v_m = -2\,(h_m\varepsilon_{13}^{(m)} + h_{m+1}\varepsilon_{13}^{(m+1)}) - \delta_{1m}h_1\,(a_{55}^{(1)}p_1^- + a_{45}^{(1)}p_2^-) -$$
$$- \delta_{N-1,m}h_N\,(a_{55}^{(N)}p_1^+ + a_{45}^{(N)}p_2^+) - 10\,(h_m\varkappa_5^{(m)} + h_{m+1}\varkappa_5^{(m+1)}) + 12\,(h_m\omega_5^{(m)} + h_{m+1}\overline{\omega}_5^{(m+1)})$$
$$v_{N-1+m} = -2\,(h_m\varepsilon_{23}^{(m)} + h_{m+1}\varepsilon_{23}^{(m+1)}) - \delta_{1m}h_1\,(a_{45}^{(1)}p_1^- + a_{44}^{(1)}p_2^-) -$$
$$- \delta_{N-1,m}h_N\,(a_{45}^{(N)}p_1^+ + a_{44}^{(N)}p_2^+) - 10\,(h_m\varkappa_4^{(m)} + h_{m+1}\varkappa_4^{(m+1)}) + 12\,(h_m\omega_4^{(m)} + h_{m+1}\overline{\omega}_4^{(m+1)}) \qquad (3.5)$$
$$\omega_4^{(m)} = \frac{2}{h_m}\alpha_4^{(m)}\int\limits_{\delta_{m-1}}^{\delta_m}\theta M_m(z)\,dz, \quad \overline{\omega}_4^{(m+1)} = \frac{2}{h_{m+1}}\alpha_4^{(m+1)}\int\limits_{\delta_m}^{\delta_{m+1}}\theta\overline{M}_{m+1}(z)\,dz$$

$$\varkappa_4^{(k)} = \alpha_4^{(k)}\int\limits_{\delta_{k-1}}^{\delta_k}\theta f_k(z)\,dz \quad (4 \rightleftarrows 5)$$

where $k = 1, 2, ..., N$; $m = 1, 2, ..., N-1$; $n = 1, 2, ..., N-2$, and the elements of the matrix G that are not defined in (3.5) are considered to be zero.

We solve the system of equations (3.5), expressing the interlayer stresses $\tau_i^{(1)}, \tau_i^{(2)}, ..., \tau_i^{(N-1)}$ through the generalized displacements u_i, w, $\beta_i^{(k)}$. Then we obtain from the integral elasticity relations (2.6)

$$\mu_1^{(k)} = q_{44}^{(k)}\,(\varepsilon_{13}^{(k)} - \varkappa_5^{(k)}) - q_{45}^{(k)}\,(\varepsilon_{23}^{(k)} - \varkappa_4^{(k)}) - 5/12\,h_k\,(\tau_1^{(k-1)} + \tau_1^{(k)}) \quad (1 \rightleftarrows 2;\ 4 \rightleftarrows 5) \qquad (3.6)$$

where $q_{mn}^{(k)}$ are the transverse shear coefficients

$$q_{mn}^{(k)} = \frac{5}{6}\,\frac{h_k a_{mn}^{(k)}}{a_{44}^{(k)}a_{55}^{(k)} - (a_{45}^{(k)})^2} \quad (m, n = 4, 5)$$

We see that the integral elasticity relations (2.6) play a fundamentally important role in the examined variant of the theory of thermoelastic multilayer shells, being the connecting link between the independent kinematic (2.1), (2.2) and static (2.3) hypotheses, since it is with their aid that we can find the relationship between the redundant functions $\tau_i^{(1)}, \tau_i^{(2)}, ..., \tau_i^{(N-1)}, \mu_i^{(1)}, \mu_i^2, ..., \mu_i^{(N)}$, introduced into (2.3), and the generalized displacements u_i, w, $\beta_i^{(k)}$.

Substituting the transverse tangential stresses (2.3) into the formula for the transverse forces (3.2) and integrating with account for the relations (3.6), we find

$$Q_1^{(k)} = q_{44}^{(k)}\,(\varepsilon_{13}^{(k)} - \varkappa_5^{(k)}) - q_{45}^{(k)}\,(\varepsilon_{23}^{(k)} - \varkappa_4^{(k)}) + 1/12\,h_k\,(\tau_1^{(k-1)} + \tau_1^{(k)})$$
$$Q_1 = \sum_{k=1}^{N} Q_1^{(k)} \quad (1 \rightleftarrows 2,\ 4 \rightleftarrows 5)$$

In conclusion we shall present the formulas for calculating the transverse normal stresses. Neglecting in formulas (2.16), (2.17) the terms k_iz in comparison with unity and integrating with respect to the coordinate z with account for the elasticity relations (3.3), the static hypothesis (2.3), and the deformation relations (3.1), we obtain

$$\sigma_{33}^{(k)} = q^- - \frac{1}{A_1A_2}\left(\frac{\partial\,(A_2N_1^{(k)}\,[z])}{\partial\alpha_1} + \frac{\partial\,(A_1N_2^{(k)}\,[z])}{\partial\alpha_2}\right) + k_1T_{11}^{(k)}\,[z] + k_2T_{22}^{(k)}\,[z]$$
$$N_1^{(k)}\,[z] = Q_1^{(k)}\,[z] - \theta_1T_{11}^{(k)}\,[z] - \theta_2T_{12}^{(k)}\,[z]$$
$$Q_1^{(k)}\,[z] = \sum_{r=1}^{k-1} Q_1^{(r)} + h_kM_k(z)\,[1 - \frac{1}{2}M_k(z)]\,\tau_1^{(k-1)} +$$
$$+ 1/2\,h_kM_k^2(z)\,\tau_1^{(k)} + M_k^2(z)\,[3 - 2M_k(z)]\,\mu_1^{(k)} \quad (1 \rightleftarrows 2)$$

$$\left\| \begin{matrix} T_{11}^{(k)}[z] \\ T_{22}^{(k)}[z] \\ T_{12}^{(k)}[z] \end{matrix} \right\| = \mathrm{A}^{(k)}[z] \left\| \begin{matrix} E_{11} \\ E_{22} \\ E_{12} \end{matrix} \right\| + \sum_{n=1}^{N} \mathrm{D}^{(kn)}[z] \left\| \begin{matrix} K_{11}^{(n)} \\ K_{22}^{(n)} \\ K_{12}^{(n)} \end{matrix} \right\| + \left\| \begin{matrix} T_{\theta 1}^{(k)}[z] \\ T_{\theta 2}^{(k)}[z] \\ T_{\theta 6}^{(k)}[z] \end{matrix} \right\|$$

$$\mathrm{A}^{(k)}[z] = \sum_{r=1}^{k-1} h_r \mathrm{b}^{(r)} + (z - \delta_{k-1}) \mathrm{b}^{(k)}$$

$$\mathrm{D}^{(kn)}[z] = \sum_{r=1}^{k-1} \left(\pi_{rn} + \frac{1}{2} \delta_{rn} h_r\right) h_r \mathrm{b}^{(r)} + \left[\pi_{kn} + \frac{1}{2} \delta_{kn} (z - \delta_{k-1})\right] (z - \delta_{k-1}) \mathrm{b}^{(k)}$$

$$T_{\theta m}^{(k)}[z] = \sum_{n=1}^{k-1} T_{\theta m}^{(n)} + \int_{\delta_{k-1}}^{z} t_{\theta m}^{(k)} dz \quad (m = 1, 2, 6)$$

Thus, we have constructed a consistent (from the viewpoint of the mixed variational principle) geometrically nonlinear variant of the theory of thermoelastic thin multilayer anisotropic shells with account for the local effects, making it possible to examine the quite general case of thermal and mechanical loadings (including tangential).

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